MultiCriteria Decision Aid with
PROMETHEE & GAIA

How to make better and more sustainable decisions

Πανεπιστήμιο Μακεδονίας
Θεσσαλονίκη, May 20-21, 2016

Bertrand MARESCHAL
Solvay Brussels School of Economics & Management
bmaresc@ulb.ac.be
http://www.promethee-gaia.net

Course Schedule

• Friday 20 – What? Why? How?
  – Morning
    • What is multicriteria decision aid
    • Why you should use multicriteria decision aid
  – Afternoon
    • PROMETHEE & GAIA

• Saturday 21 – Using Visual PROMETHEE
  – Morning
    • Hands-on training
  – Afternoon
    • Case studies
Friday 20 – What? Why? How?

• **Morning**
  - What is multicriteria decision aid
    • Optimization vs multicriteria decision aid
    • Sustainable decisions
    • Models and main approaches
  - Why you should use multicriteria decision aid
    • Facing complexity and conflicting issues
    • Single decision maker context
    • Collaborative group decision
    • Negotiations between parties or stakeholders

• **Afternoon**
  - PROMETHEE & GAIA

Decisions

• **Personal decisions**
  - Choose a restaurant tonight
  - Choose a university
  - Purchase a new phone, a new car, ...

• **Business decisions**
  - Develop a new product
  - Choose a computer system
  - Investments, strategies, project management, ...

• **Political decisions**
  - Join the EU... Leave the EU...
  - Build a new hospital
  - Regional investment, taxes, ...
Decision Making

- Describe,
- Understand,
- Manage.

2 Approaches:
- Qualitative approach,
- Quantitative approach.

Real World
- Social
- Political
- Economical
- Industrial
- Environmental
- Military

Decision Aid

- Possible decisions?
- How to compare them?
- Preferences, Objectives?
Decision Aid

Real World
- Social
- Political
- Economical
- Industrial
- Environmental
- Military

Quantitative model

- Approximation to real world!
  - Decision Aid.

Quantitative Model?

- Describe possible decisions (actions)?
  - List, variables, ...
- Objective? Best decision? Best choice?
  - Minimize costs?
  - Maximize profit?
  - Maximize quality?
  - Minimize impacts?
- Optimization models?
- Or MCDA (MultiCriteria Decision Aid) models...
MCDA vs Optimisation

- **Optimisation**
  - Single criterion: Profit
  - Short-sightedness
  - Crisis

- **MCDA**
  - Profit, Social, Environment, ...
  - Sustainable Decisions

**Some Decision or Evaluation Problems**

- Locating a new plant, a new shop, ...
- Human resources management.
- Purchasing equipment.
- Assessing the quality of suppliers.
- Evaluating projects.
- Selecting an investment strategy.
Unicriterion vs Multicriteria Model

• Unicriterion model:

\[ \text{Optimise}\{ g(a) | a \in A \} \]

– Mathematically well-stated:
  • Optimal solution,
  • Complete ranking of the actions.

– Socio-economically ill-stated:
  • Single criterion? Not realistic.
  • Notion of criterion: perception thresholds, ...

Unicriterion vs Multicriteria Model

• Multicriteria model:

\[ \text{Optimise}\{ g_1(a), g_2(a), ..., g_k(a) | a \in A \} \]

– Mathematically ill-stated:
  • No optimal solution,
  • No mathematical meaning.

– Socio-economically well-stated:
  • Closer to real world decision problem,
  • Search for a compromise solution.
Multicriteria Table

- **Actions:**
  - Possible decisions,
  - Items to evaluate.
- **Criteria:**
  - Quantitative,
  - Qualitative.

| Action | Crit. 1 (20) | Crit. 2 (rating) | Crit. 3 (qual.) | Crit. 4 (Y/N) | ...
|--------|-------------|-----------------|----------------|-------------|---
| 1      | 18          | 135             | G              | Yes         | ...
| 2      | 9           | 147             | B              | Yes         | ...
| 3      | 15          | 129             | VG             | No          | ...
| 4      | 12          | 146             | VB             | ?           | ...
| 5      | 7           | 121             | G              | Yes         | ...
| ...    | ...         | ...             | ...            | ...         | ...

Θεσσαλονίκη, May 20-21, 2016
### Plant Location

<table>
<thead>
<tr>
<th></th>
<th>Investment (M€)</th>
<th>Costs (k€)</th>
<th>Environm. (impact)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Site 1</td>
<td>18</td>
<td>135</td>
<td>G</td>
<td>...</td>
</tr>
<tr>
<td>Site 2</td>
<td>9</td>
<td>147</td>
<td>B</td>
<td>...</td>
</tr>
<tr>
<td>Site 3</td>
<td>15</td>
<td>129</td>
<td>VG</td>
<td>...</td>
</tr>
<tr>
<td>Site 4</td>
<td>12</td>
<td>146</td>
<td>VB</td>
<td>...</td>
</tr>
<tr>
<td>Site 5</td>
<td>7</td>
<td>121</td>
<td>G</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Θεσσαλονίκη, May 20-21, 2016

### Purchase Options

<table>
<thead>
<tr>
<th></th>
<th>Price (€)</th>
<th>Reliability (days)</th>
<th>Maintenance (estimate)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Product A</td>
<td>18</td>
<td>135</td>
<td>G</td>
<td>...</td>
</tr>
<tr>
<td>Product B</td>
<td>9</td>
<td>147</td>
<td>B</td>
<td>...</td>
</tr>
<tr>
<td>Product C</td>
<td>15</td>
<td>129</td>
<td>VG</td>
<td>...</td>
</tr>
<tr>
<td>Product D</td>
<td>12</td>
<td>146</td>
<td>VB</td>
<td>...</td>
</tr>
<tr>
<td>Product E</td>
<td>7</td>
<td>121</td>
<td>G</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Θεσσαλονίκη, May 20-21, 2016
A Simple Example

The purchase of a new car

Objectives:
• Economy (price),
• Usage (fuel consumption),
• Performance (power),
• Space,
• Comfort.

Multicriteria Table

<table>
<thead>
<tr>
<th>Cars</th>
<th>Price</th>
<th>Power</th>
<th>Consumpt.</th>
<th>Space</th>
<th>Comfort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tourism A</td>
<td>26 000 $</td>
<td>75</td>
<td>8.0</td>
<td>average</td>
<td>average</td>
</tr>
<tr>
<td>Sport</td>
<td>29 000 $</td>
<td>110</td>
<td>9.0</td>
<td>very bad</td>
<td>bad</td>
</tr>
<tr>
<td>Tourism B</td>
<td>25 500 $</td>
<td>85</td>
<td>7.0</td>
<td>good</td>
<td>average</td>
</tr>
<tr>
<td>Luxury 1</td>
<td>38 000 $</td>
<td>90</td>
<td>8.5</td>
<td>good</td>
<td>very good</td>
</tr>
<tr>
<td>Economic</td>
<td>15 000 $</td>
<td>50</td>
<td>7.5</td>
<td>bad</td>
<td>very bad</td>
</tr>
<tr>
<td>Luxury 2</td>
<td>35 000 $</td>
<td>85</td>
<td>9.0</td>
<td>very good</td>
<td>good</td>
</tr>
</tbody>
</table>

• Best buy?
• Best compromise?
• Priorities of the buyer?
1. Defining the actions

• Definition: Let \( A \) the set of actions. \( A \) can be defined:
  – in extension: by enumeration of its elements.
    - relatively small number of actions.
  – in comprehension: by constraints on a set of decision variables.
    (Cf. linear programming)
    - large number or infinity of actions.
Some properties of the set of actions

\( \mathcal{A} \) can be:

- **stable**: a priori defined, doesn’t evolve.
- **evolutive**: can evolve during the procedure.
- **globalised**: mutually exclusive elements.
- **fragmented**: combinations of actions are considered.

2. Defining the criteria

- **Definition**: function \( g \) defined on \( \mathcal{A} \), taking its values in a totally ordered set, and representing an objective of the decision-maker.
- **Consistent family of criteria**:
  - Include all aspects of the decision problem, all the objectives of the decision-maker,
  - Avoid redundancies.
Qualitative Criteria

• Qualitative scales:
  – Maximum 9 levels (7 ± 2) to ensure a consistent evaluation.
  – Presence of a neutral level?
  – Examples:
    • Very good, Good, Average, Bad, Very bad
    • Yes, No
    • ++, +, 0, -, --
    • ++, +, -, --
• Underlying numerical scale (coding).

3. Modeling preferences

• Problem:
  How to compare two actions $a$ and $b$ to each other?
• A first model: 3 possible results:
  1. Preference: $aPb$ or $bPa$
  2. Indifference: $aIb$
  3. Incomparability: $aRb$
Traditional preference structure (unicriterion)

- Optimisation of a function \( g \) on \( A \)
  \[
  \forall a, b \in A: \begin{cases} 
  aPb & \iff g(a) > g(b) \\
  aIb & \iff g(a) = g(b)
  \end{cases}
  \]

- Consequences:
  - \( R \) is empty
  - \( P \) is transitive
  - \( I \) is transitive

- Complete ranking.

The notion of indifference threshold

- Problem: Indifference can be intransitive.
  - Cf. Coffee cup paradox (Luce, 1956)
The notion of indifference threshold

• Problem: Indifference can be intransitive.
  Cf. Coffee cup paradox (Luce, 1956)

• Introduction of an indifference threshold:

\[ \forall a, b \in A : \begin{cases} 
  aPb & \iff g(a) > g(b) + q \\
  aIb & \iff |g(a) - g(b)| \leq q 
\end{cases} \]

• Quasi-order: \( P \) is transitive, but not \( I \).

Other preference structures

• Variable indifference threshold
  \( \succ \) Interval order.

• Preference + indifference thresholds
  \( \succ \) Pseudo-order.

• Models including incomparability
  \( \succ \) Partial orders.

• Valued preference structures.
Problematics

- **α - choice**: determine a subset of actions (the « best ones »).
- **β - sorting**: sort actions in predefined categories.
- **γ - ranking**: rank from the best to the worst action.
- **δ - description**: describe actions and their consequences.

<table>
<thead>
<tr>
<th></th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$g_3$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$g_1(a)$</td>
<td>$g_2(a)$</td>
<td>$g_3(a)$</td>
<td>...</td>
</tr>
<tr>
<td>$b$</td>
<td>$g_1(b)$</td>
<td>$g_2(b)$</td>
<td>$g_3(b)$</td>
<td>...</td>
</tr>
<tr>
<td>$c$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Evaluations

- **n actions**
- **k criteria**

Unicriterion Model Optimization

- **Optimal solution**
  - Maximum or minimum of criterion value
  - Mathematically well-defined
  - Generally exists
- **Optimization algorithms**
  - Adapted to specific cases (linear programming, branch and bound, network optimization, ...)
  - ”Proved”
Multicriteria Model
Dominance and efficiency

• « Objective ».
• Based on a unanimity principle:

\[ a \text{ dominates } b \iff g_h(a) \geq g_h(b) \quad \forall h \]

• Efficiency: \( a \) is efficient if it is not dominated by any other action.
• Problems:
  – Dominance is poor (few dominances),
  – Many actions are efficient.

Objections to Dominance

<table>
<thead>
<tr>
<th></th>
<th>( g_1 )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \( a \) efficient
- \( a \) preferred to \( b \)

<table>
<thead>
<tr>
<th></th>
<th>( g_1 )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>100</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \( a \) and \( b \) efficient
- \( a \) and \( b \) indiff.

<table>
<thead>
<tr>
<th></th>
<th>( g_1 )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>100</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \( a \) and \( b \) efficient
- \( a \) preferred to \( b \)

<table>
<thead>
<tr>
<th></th>
<th>( g_1 )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV</td>
<td>100</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>99</td>
<td>100</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \( a \) and \( b \) efficient
- \( a \) and \( b \) indiff.

<table>
<thead>
<tr>
<th></th>
<th>( g_1 )</th>
<th>( g_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>99</td>
<td>99</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \( a \) efficient
- \( a \) and \( b \) indiff.
Some Characteristics for a good multicriteria method

- Take into account **deviations** between evaluations.
- Take **scale** effects into account.
- Build either a **partial** \((P,I,R)\) or a **complete** \((P,I)\) ranking of the actions.
- Stay sufficiently **simple**:  
  - no black box,  
  - no technical parameters.

A common approach:

The weighted sum

<table>
<thead>
<tr>
<th>Actions or Decisions</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(g_1(a)g_2(a)g_3(a)) ...</td>
</tr>
<tr>
<td>(b)</td>
<td>(g_1(b)g_2(b)g_3(b)) ...</td>
</tr>
<tr>
<td>(c)</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>(w_1)</td>
<td>(w_2)</td>
</tr>
</tbody>
</table>
A common approach:
The weighted sum

- Global value for $a$:
  \[ V(a) = w_1 g_1(a) + w_2 g_2(a) + \ldots \]

- $a$ is preferred to $b$ if:
  \[ V(a) > V(b) \]
  (if all criteria are to maximise)

**Weighted Sum Example 1**

<table>
<thead>
<tr>
<th></th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$g_3$</th>
<th>$g_4$</th>
<th>$g_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>55</td>
</tr>
<tr>
<td>$b$</td>
<td>85</td>
<td>85</td>
<td>85</td>
<td>85</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>$1/5$</td>
<td>$1/5$</td>
<td>$1/5$</td>
<td>$1/5$</td>
<td>$1/5$</td>
</tr>
</tbody>
</table>

- $V(a) = 91$      \[ V(b) = 88 \]
- Total and uncontrolled compensation of weaknesses by strengths.
### Weighted Sum Example 2

<table>
<thead>
<tr>
<th></th>
<th>$g_1$</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>$b$</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>$c$</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$d$</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

$\frac{1}{2} \quad \frac{1}{2}$

- $V(a) = V(b) = V(c) = V(d) = 50$
- Elimination of conflicts – Loss of information.

---

### Weighted Sum Example 3

"Profit is approximately 2 times more important than time savings; 0.7 for profit and 0.3 for time savings."

<table>
<thead>
<tr>
<th></th>
<th>$g_1$ (BF)</th>
<th>$g_2$ (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>$b$</td>
<td>48</td>
<td>70</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c|cc}
 & V(a) & V(b) \\
\hline
a & 60 & 54.6 \\
\end{array}
\]

$a$ is ranked 1\textsuperscript{st}.
**Weighted Sum**

**Example 3**

“Profit is approximately 2 times more important than time savings; 0.7 for profit and 0.3 for time savings.”

<table>
<thead>
<tr>
<th></th>
<th>$g_1$ (FF)</th>
<th>$g_2$ (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>$b$</td>
<td>8</td>
<td>70</td>
</tr>
</tbody>
</table>

$V(a) = 25$

$V(b) = 26.6$

$b$ is ranked 1st!

---

**Weighted Sum**

**Example 3**

<table>
<thead>
<tr>
<th></th>
<th>$g_1$ (BF)</th>
<th>$g_2$ (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>$b$</td>
<td>48</td>
<td>70</td>
</tr>
</tbody>
</table>

$V(a) = 60$

$V(b) = 54.6$

$a$ is ranked 1st.

<table>
<thead>
<tr>
<th></th>
<th>$g_1$ (FF)</th>
<th>$g_2$ (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>$b$</td>
<td>8</td>
<td>70</td>
</tr>
</tbody>
</table>

$V(a) = 25$

$V(b) = 26.6$

$b$ is ranked 1st.

*Significance of the “weights”!*
Multicriteria Decision Aid

- Multiattribute utility theory (MAUT).
- Outranking methods.
- Interactive methods.
- Multiobjective programming.
- ...

Since 1970, numerous developments: conferences, papers, books, applications, software...

Multiatribute Utility Theory

- Single synthesis criterion (aggregation).

\[ U(a) = U(g_1(a), g_2(a), \ldots, g_k(a)) \]

- Existence?
- Construction?
- Mathematical form?
- \( \Rightarrow \) additive?

\[ U(a) = \sum_{j=1}^{k} U_j(g_j(a)) \]
Multiattribute Utility Theory

- Mode of construction:
  - direct,
  - indirect.
- Information intensive for the decision maker.
  (quantity of information vs reliability?).
- Not flexible (sensitivity analyses).
- Far away from the original decision problem structure:
  multicriteria \(\Rightarrow\) unicriterion

Outranking Methods

- Majority principle
  (vs unanimity for dominance).
- Pairwise comparison of actions.
- Closer to the decision problem.
- **ELECTRE** methods (1968-).
- **PROMETHEE & GAIA** methods (1982-).
Friday 20 – What? Why? How?

• Morning
  – What is multicriteria decision aid
  – Why you should use multicriteria decision aid

• Afternoon
  – PROMETHEE & GAIA
    • Data and preference modeling
    • PROMETHEE rankings
    • GAIA visual analysis
    • Sensitivity analyses
    • Group decision and negotiation

Decision Aid Methods

• Preference modelling:
  Perception of scales
  Weighing of criteria

• Analysis Procedure:
  Prescriptive approach: PROMETHEE
  Descriptive approach: GAIA
**Why PROMETHEE?**

- Proven methodology
  - 30 years development
  - Over 1350 published scientific papers
- « Simplicity »
- Visual tools
- Sensitivity analysis tools
- Interactivity
- **Visual PROMETHEE software**

---

**Some stats...**

- Over **1350** published papers as of today.
- **81** papers published by **140** Greek authors, from **1989** to **2016**
  (worldwide #4, after Belgium, China and Brazil)
- Main fields of application:
  - Environment
  - Industry
  - Services / Public sector
  - Energy
  - Finance

---
PROMETHEE Timeline

- Over 1350 papers published
  - 75% applied – 25% theoretical
- Median year: 2011
- Over 2300 authors from 78 countries

Principles of the PROMETHEE Methods

- Preference modelling:
  - Preference functions
  - Weighing of the criteria
- Pairwise comparison of the actions:
  - Outranking
  - Prudent (partial ranking)
  - Partially compensatory approach
    - Advantage over weighted sum and utility functions
### Comparison of 2 Actions

<table>
<thead>
<tr>
<th></th>
<th>Crit. 1 (/20)</th>
<th>Crit. 2 (rating)</th>
<th>Crit. 3 (qual.)</th>
<th>Crit. 4 (Y/N)</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action 1</td>
<td>18</td>
<td>135</td>
<td>G</td>
<td>Yes</td>
<td>...</td>
</tr>
<tr>
<td>Action 2</td>
<td>9</td>
<td>147</td>
<td>Difference = 6</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>Action 3</td>
<td>15</td>
<td>129</td>
<td>VG</td>
<td>No</td>
<td>...</td>
</tr>
<tr>
<td>Action 4</td>
<td>12</td>
<td>146</td>
<td>VB</td>
<td>?</td>
<td>...</td>
</tr>
<tr>
<td>Action 5</td>
<td>7</td>
<td>121</td>
<td>G</td>
<td>Yes</td>
<td>...</td>
</tr>
</tbody>
</table>

### Preference Function

#### Linear Q

- Preference degree
- Indifference threshold
- Preference threshold
- Difference
PROMETHEE

<table>
<thead>
<tr>
<th>Economic</th>
<th>Pref (Eco.,Lux.)</th>
<th>Pref (Lux.,Eco.)</th>
<th>Pref (Lux.,Eco.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>23000</td>
<td>0,0</td>
<td>0,0</td>
</tr>
<tr>
<td>Power</td>
<td>60</td>
<td>0,0</td>
<td>1,0</td>
</tr>
<tr>
<td>Fuel</td>
<td>7,5</td>
<td>-0,5</td>
<td>0,0</td>
</tr>
<tr>
<td>Space</td>
<td>B</td>
<td>0,5</td>
<td>50</td>
</tr>
<tr>
<td>Comfort</td>
<td>VB</td>
<td>0,0</td>
<td>1,0</td>
</tr>
</tbody>
</table>

Preference Deviation:

- Pref (Eco.,Lux.) = 0,3
  \[= \frac{1 + 0 + 0,5 + 0 + 0}{5}\]
- Pref (Lux.,Eco.) = 0,5
  \[= \frac{0 + 1 + 0 + 0,5 + 1}{5}\]

Θεσσαλονίκη, May 20-21, 2016
Pairwise Comparisons

- For each criterion $g_j$:
  - Preference function $P_j$
  - Weight $w_j$
- Multicriteria preference degree of $a$ over $b$:

$$\pi(a,b) = \sum_{j=1}^{k} w_j P_j(a,b)$$
### Pairwise comparisons

#### Preference Deviation

<table>
<thead>
<tr>
<th></th>
<th>Pref (Eco., Lux.)</th>
<th>Pref (Lux., Eco.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eco.</td>
<td>Lux.</td>
<td>Lux.</td>
</tr>
<tr>
<td></td>
<td>Pref</td>
<td>Pref</td>
</tr>
<tr>
<td>Eco.</td>
<td>-23000</td>
<td>0.00</td>
</tr>
<tr>
<td>Lux.</td>
<td>15000</td>
<td>38000</td>
</tr>
<tr>
<td></td>
<td>Price</td>
<td>90</td>
</tr>
<tr>
<td>Eco.</td>
<td>100</td>
<td>0.5</td>
</tr>
<tr>
<td>Lux.</td>
<td>0.0</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>Power</td>
<td>Fuel</td>
</tr>
<tr>
<td>Eco.</td>
<td>0.5</td>
<td>G</td>
</tr>
<tr>
<td>Lux.</td>
<td>2.5</td>
<td>VB</td>
</tr>
<tr>
<td></td>
<td>Comfort</td>
<td>Comfort</td>
</tr>
</tbody>
</table>

#### Computation of Preference Flows

<table>
<thead>
<tr>
<th></th>
<th>Tour.A</th>
<th>Sport</th>
<th>Tour.B</th>
<th>Lux.1</th>
<th>Econ.</th>
<th>Lux.2</th>
<th>( \phi (a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tour.A</td>
<td>0.00</td>
<td>0.34</td>
<td>0.00</td>
<td>0.21</td>
<td>0.26</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>Sport</td>
<td>0.20</td>
<td>0.00</td>
<td>0.16</td>
<td>0.24</td>
<td>0.30</td>
<td>0.24</td>
<td>0.23</td>
</tr>
<tr>
<td>Tour.B</td>
<td>0.15</td>
<td>0.55</td>
<td>0.00</td>
<td>0.32</td>
<td>0.45</td>
<td>0.33</td>
<td>0.36</td>
</tr>
<tr>
<td>Lux.1</td>
<td>0.18</td>
<td>0.45</td>
<td>0.10</td>
<td>0.00</td>
<td>0.50</td>
<td>0.15</td>
<td>0.28</td>
</tr>
<tr>
<td>Econ.</td>
<td>0.20</td>
<td>0.34</td>
<td>0.14</td>
<td>0.30</td>
<td>0.00</td>
<td>0.35</td>
<td>0.27</td>
</tr>
<tr>
<td>Lux.2</td>
<td>0.24</td>
<td>0.30</td>
<td>0.10</td>
<td>0.04</td>
<td>0.60</td>
<td>0.00</td>
<td>0.26</td>
</tr>
<tr>
<td>( \phi (a) )</td>
<td>0.19</td>
<td>0.40</td>
<td>0.10</td>
<td>0.22</td>
<td>0.42</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>( \phi(a) )</td>
<td>0.02</td>
<td>-0.17</td>
<td>0.26</td>
<td>0.06</td>
<td>-0.15</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>
Preference Flows

- Relative scores computed for the actions.
- Leaving (+) and entering (−) flows:
  - Strength: \(0 \leq \phi^+ \leq 1\)
  - Weakness: \(0 \leq \phi^- \leq 1\)
- Net flow:
  - Balance: \(-1 \leq \phi = \phi^+ - \phi^- \leq +1\)
- Unicriterion net flows:
  - Standardized scores for each criterion:
    
  criterion \(f_j\) \(\Rightarrow\) \(-1 \leq \phi_j \leq +1\)

PROMETHEE I & II

- **PROMETHEE I**: partial ranking – \(\phi^+, \phi^-\)
- **PROMETHEE II**: complete ranking – \(\phi\)
Sensitivity Analysis with PROMETHEE

• Criteria weights \( \Rightarrow \) PROMETHEE ranking.
• Interactive weight sensitivity analysis: « Walking Weights ».
• Robustness with respect to weight values?
  – Weight stability intervals.
  – Visual weight stability intervals.
Walking Weights

Visual Stability Intervals

VSI for « Price » (level 6):
[19.20%, 23.70%]
Limits of a Ranking Method

- Robustness of the ranking?
- « Blind » sensitivity analysis.
- Closely ranked actions can have quite different profiles.
- Origin of incomparabilities?
  ➢ Usefulness of a complementary descriptive approach.

Properties of the Net Flow

- Net flow is centered: \[ \sum_{a \in A} \phi(a) = 0 \]
- Unicriterion net flows:
  \[ \phi(a) = \sum_{j=1}^{k} w_j \cdot \phi_j(a) \]
  with
  \[ \phi_j(a) = \frac{1}{n-1} \sum_{b \in A} \left[ P_j(a,b) - P_j(b,a) \right] \]
GAIA

- Visual descriptive analysis.
- Better understanding:
  - Conflicting criteria.
  - Action profiles.
  - Possible compromise solutions.
- Reducing the multicriteria dimension:
  - Principal components analysis.

---

GAIA

1. Computation of unicriterion net flows (normalization)
2. Projection on a plane:

- Graphical representation.
- 5 dimensions!
• Discover conflicts among criteria.
• Identify potential compromises.
• Help to fix priorities.

• Actions: points
• Criteria: axes
Price
- Economic: 15 k€
- Tourism: 25-5-26 k€
- Sport: 29 k€
- Luxury: 35-38 k€

Power
- Sport: 110 kW
- Luxury: 85-90 kW
- Tourism: 75-85 kW
- Economic: 50 kW
### PROMETHEE II!

- Tourism B: 0.26
- Luxury 1: 0.06
- Tourism A: 0.02
- Luxury 2: 0.00
- Economic: -0.15
- Sport: -0.17

**Decision axis**

- only 90% information!!
Sensitivity Analysis with GAIA

- Criteria weights ➔ Decision axis position.
- Interactive weight sensitivity analysis: « Walking Weights ».
- Robustness with respect to weight values?
  - Decision maker « brain » (PROMETHEE VI).
  - Area determined by the tip of the decision axis when criteria weights are changed within predefined percentages.

GAIA-Brain

20 years old

35 years old

Θεσσαλονίκη, May 20-21, 2016
Enhancing GAIA

• Limits of GAIA:
  – Imperfect view of the multicriteria data ($\Delta\%$).
  – Potential distortion of the action profiles.
  – Non-optimal representation of the decision axis (weights) and distortion of the PROMETHEE II ranking (especially when the decision axis is shorter).

• New « GAIA-type » views:
  – GAIA 3D
  – GAIA Webs
One or Several Stakeholders?

- **Single stakeholder:**
  - One actor (decision maker)
  - One multicriteria table and one preference structure
- **Multiple stakeholders:**
  - Several actors (including decision maker(s))
  - Several multicriteria tables and preference structures
  - Search for consensus
Multi-scenarios Model

• Scenarios:
  – Points of view,
  – Hypotheses, ...

• Evaluations:
  – ‘Objective’ criteria: common evaluations.
  – ‘Subjective’ criteria: specific evaluations for each scenario.

• Specific preference structures:
  – Weights, preference thresholds.

• Adaptation of PROMETHEE:
  – Individual rankings
  – Global (group) ranking with possible weighing of the scenarios

• Adaptation of GAIA:
  – Three different analyses:
    • GAIA-Criteria
    • GAIA-Scenarios
    • GAIA-Unicriterion
Example

- A Greek family
- Three actors ("decision makers"):  
  - Jason (dad),
  - Danae (mom),
  - George (the kid).
- Three scenarios.
- Three multicriteria tables:
  - Different weights.
  - Subjective evaluation of comfort.

Individual PROMETHEE rankings

[Diagram showing individual PROMETHEE rankings]

Example

- A Greek family
- Three actors ("decision makers"):  
  - Jason (dad),
  - Danae (mom),
  - George (the kid).
- Three scenarios.
- Three multicriteria tables:
  - Different weights.
  - Subjective evaluation of comfort.

Individual PROMETHEE rankings

[Diagram showing individual PROMETHEE rankings]
GDSS-GAIA: Criterion Comfort

Θεσσαλονίκη, May 20-21, 2016

« Family » (group) ranking

Θεσσαλονίκη, May 20-21, 2016
Other PROMETHEE Tools

- PROMETHEE V
  - Portfolio selection under constraints
- PROMETHEE Sort
- Bank Adviser
- PROMETHEE Efficiency Analysis
  - Input/output model
- ...

Google Maps interface
Homework Assignment

• For tomorrow afternoon.
• By small groups of 4 to 6 students.
• Set up a multicriteria decision problem:
  – Actual or fictive.
  – Including minimum 6 actions, 5 criteria and 2 scenarios.
  – Prepare the multicriteria evaluation table.
  – Think about preference functions and criteria weights.

Saturday 20 – Using Visual PROMETHEE

• Morning
  – Hands-on training with Visual PROMETHEE
    • Learn the software interface
    • Interpret results and displays
    • Practice with simple numerical examples

• Afternoon
  – Case studies
Steps for using PROMETHEE

1. Define the actions (list)
2. Define the criteria
   - Quantitative
   - Qualitative (scale)
3. Build the evaluation table.
4. For each criterion:
   - Choose the right type of preference function
   - Set the appropriate thresholds
5. Set the weights of the criteria

Preference Functions

- For continuous quantitative criteria (e.g. cost, price, power):
  - V-shape (no indifference threshold)
  - Linear
- For qualitative or discrete quantitative criteria (e.g. « very good to very bad », number of USB ports):
  - Usual (no thresholds)
  - Level
Visual PROMETHEE
WWW.PROMETHEE-GAIA.NET

- **Visual PROMETHEE** software:
  - Free Academic Edition
  - Business Edition
- [http://biblio.promethee-gaia.net](http://biblio.promethee-gaia.net)
  - Over 1350 references
- **Visual PROMETHEE** Manual (PDF or ebook)
- Services: Training, Coaching, Free seminars
- [http://blog.promethee-gaia.net](http://blog.promethee-gaia.net)
- [http://www.promethee-days.com](http://www.promethee-days.com) Spring 2017
- LinkedIn group, Twitter, ResearchGate, ...

---

Saturday 20 – Using Visual PROMETHEE

- **Morning**
  - Hands-on training with **Visual PROMETHEE**
    - Learn the software interface
    - Interpret results and displays
    - Practice with simple numerical examples
- **Afternoon**
  - Case studies